Diffraction Shaders

From microsurface models to reflection models

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Alias | wavefront
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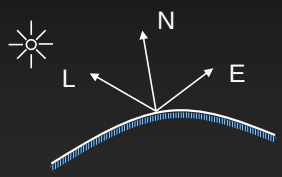
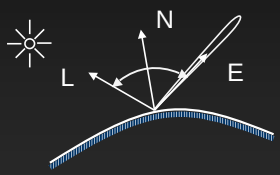
Shaders

Model reflection from surfaces



Shaders

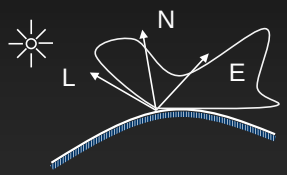
Perfectly smooth surface



Not very interesting

Shaders

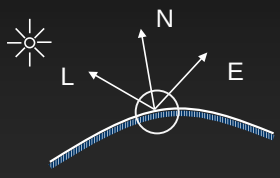
Rough surfaces



More interesting

Shaders

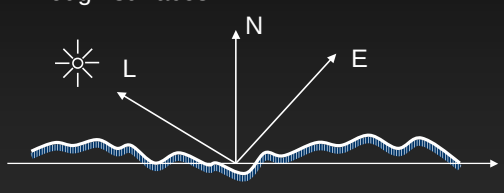
Rough surfaces



Zoom in on microstructure

Shaders

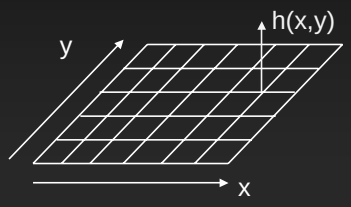
Rough surfaces



Reflection depends on the microsurface

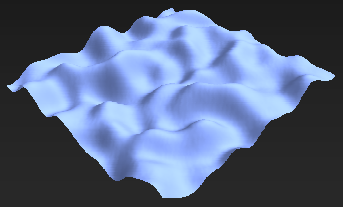
Microsurface Models

Model surface as a two-dimensional (random) height field



Microsurface Models

Isotropic Gaussian (smooth)



Microsurface Models

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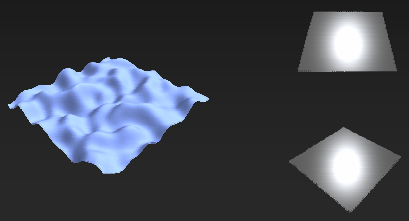
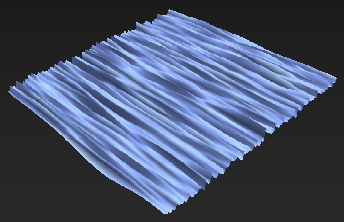


Microsurface Models

Anisotropic Gaussian (brushed metal)



Microsurface Models

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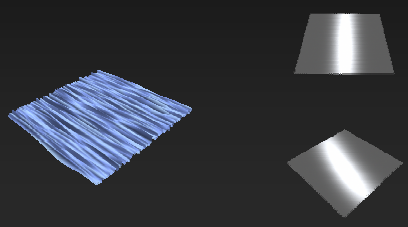
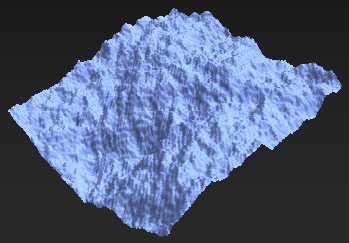


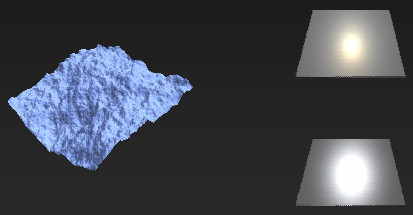
Microsurface Models

Fractal



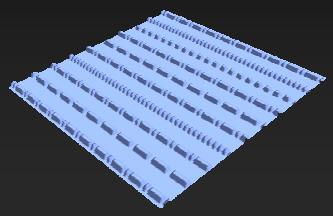
Microsurface Models

Fractal



Microsurface Models

Periodic (compact disk)



Microsurface Models

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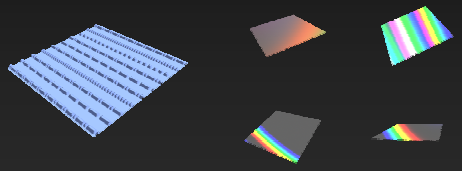


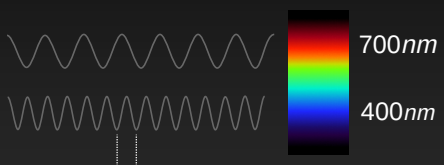
Our Approach

Use waves to model both

- the propagation of light
- the microsurface (Fourier Analysis)

Generalization of previous models

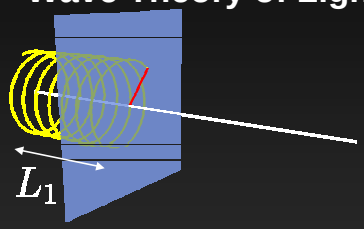
Wave Theory of Light



wavelength

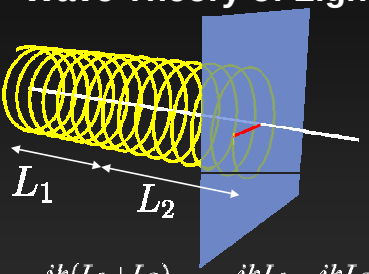
λ $k = \frac{2\pi}{\lambda}$

Wave Theory of Light



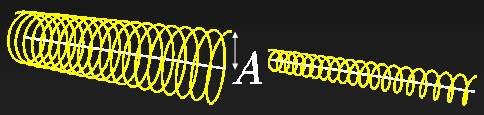
$e^{ikL_1} = \cos(kL_1) + i\sin(kL_1)$ $k = \frac{2\pi}{\lambda}$

Wave Theory of Light



$$e^{ik(L_1+L_2)} = e^{ikL_1}\ e^{ikL_2}$$

Intensity of a Wave



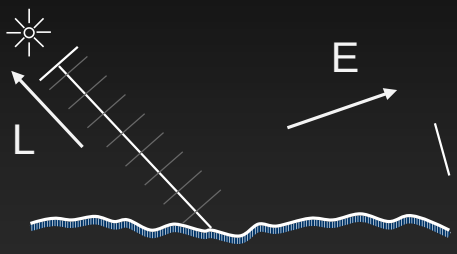
$$I = |A\ e^{ikL}|^2 = A^2$$

Diffraction

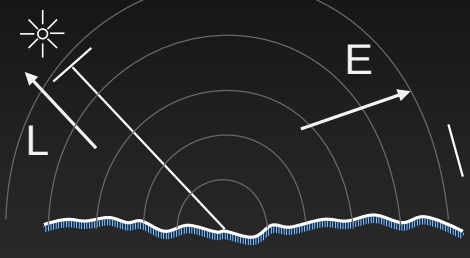
$$|A_1\ e^{ikL_1} + A_2\ e^{ikL_2}|^2 \neq A_1^2 + A_2^2$$

Intensities don't add up in general

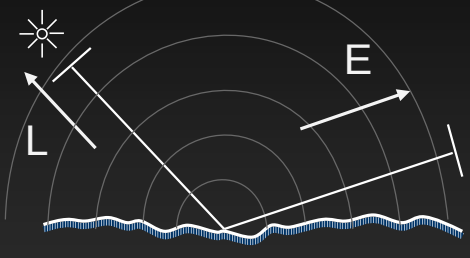
Interaction: Light-Surface



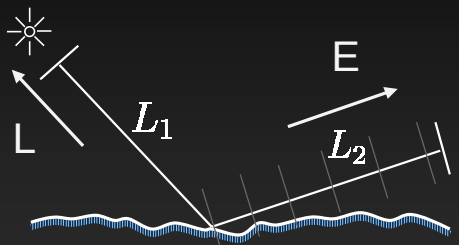
Interaction: Light-Surface



Interaction: Light-Surface

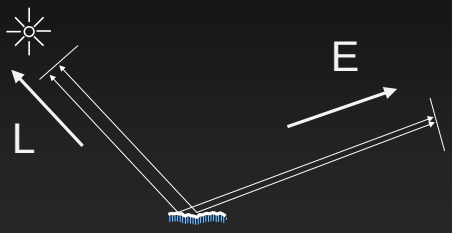


Interaction: Light-Surface



$$e^{ik(L_1+L_2)} = e^{ikL_1}\, e^{ikL_2}$$

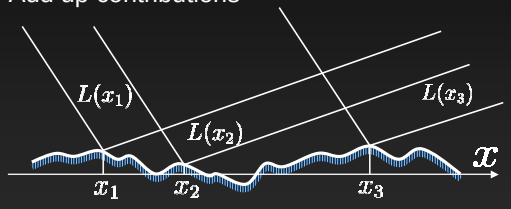
Interaction: Light-Surface



Source and observer “far away”

Interaction: Light-Surface

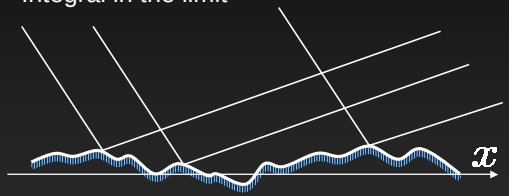
Add up contributions



$$\psi = e^{ikL(x_1)} + e^{ikL(x_2)} + e^{ikL(x_3)} + \cdots$$

Interaction: Light-Surface

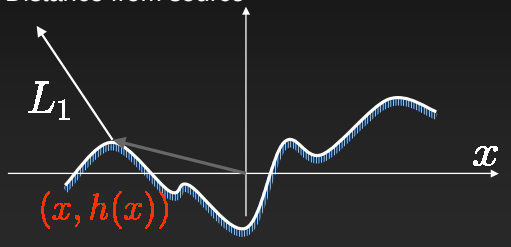
Integral in the limit



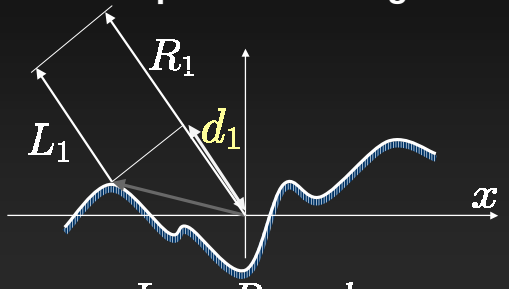
$$\psi = \int e^{ikL(x)}\, dx$$

Compute Path Length

Distance from source

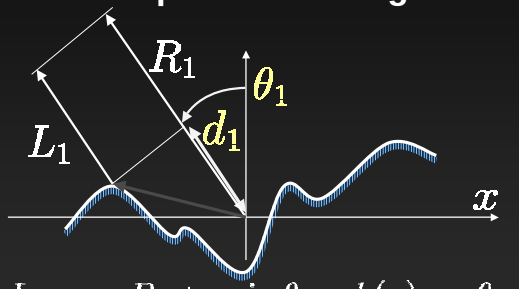


Compute Path Length



$$L_1 = R_1 - d_1$$

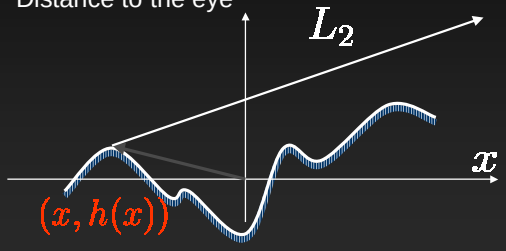
Compute Path Length



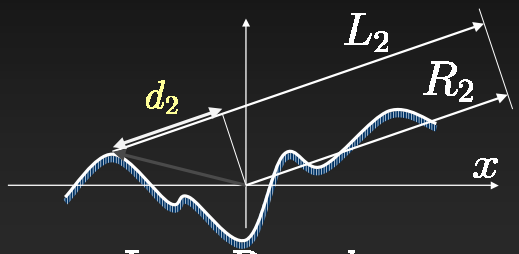
$$L_1 = R_1 + x\sin\theta_1 - h(x)\cos\theta_1$$

Compute Path Length

Distance to the eye

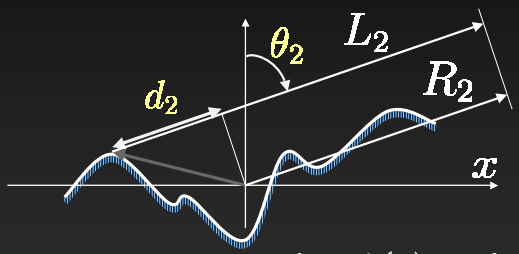


Compute Path Length



$$L_2 = R_2 + d_2$$

Compute Path Length



$$L_2 = R_2 - x\sin\theta_2 - h(x)\cos\theta_2$$

Compute Path Length

Putting the two together:

$$\begin{aligned} L_1 &= R_1 + x\sin\theta_1 - h(x)\cos\theta_1 \\ L_2 &= R_2 - x\sin\theta_2 - h(x)\cos\theta_2 \end{aligned}$$

Compute Path Length

Putting the two together:

$$\begin{aligned} L_1 &= R_1 + x\sin\theta_1 - h(x)\cos\theta_1 \\ +\ L_2 &= R_2 - x\sin\theta_2 - h(x)\cos\theta_2 \end{aligned}$$

$$L(x) = R_1 + R_2 + ux + wh(x)$$

$$\begin{aligned} u &= \sin\theta_1 - \sin\theta_2 \\ w &= -\cos\theta_1 - \cos\theta_2 \end{aligned}$$

Reflected Wave

Now integrate over the surface:

$$L(x) = \underbrace{R_1 + R_2 + ux + wh(x)}$$

$$\psi = \int e^{ikL(x)}\, dx$$

Reflected Wave

$$L(x) = \underbrace{R_1 + R_2}_{\swarrow} + ux + \underbrace{wh(x)}_{\searrow}$$

$$\psi = C \int e^{ikwh(x)}\, e^{ikux}\, dx$$

Fourier Transform

$$\boxed{\psi = C \int p(x)\, e^{ikux}\, dx}$$

$$p(x) = e^{ikwh(x)}$$

Key insight

Fourier Transform

$$\boxed{\psi = C\, P(ku)}$$

$$p(x) = e^{ikwh(x)}$$

Key insight

Fourier Transform

Simple relationship:

$$\boxed{I = |P(ku)|^2}$$

$$|C| = 1$$

Example

Smooth surface:

$$h(x) = 0$$

$$p(x) = e^{ikwh(x)} = e^0 = 1$$

$$P(ku) = \delta(ku)$$

$$u = \sin\theta_1 - \sin\theta_2$$

Example

Almost smooth surface:

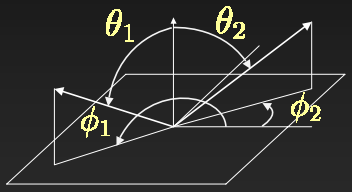
$$p(x) = e^{ikwh(x)} \approx 1 + ikwh(x)$$

$$P(ku) = \delta(ku) + ikwH(ku)$$

$$|P(ku)|^2 = \delta(ku) + k^2w^2|H(ku)|^2$$

Two dimensions

Previous derivation extends to 2D



Two dimensions

$$I = |P(ku, kv)|^2$$

$$p(x, y) = e^{ikwh(x,y)}$$

$$k = \frac{2\pi}{\lambda}$$

$$\begin{aligned} u &= -\cos\phi_1 \sin\theta_1 - \cos\phi_2 \sin\theta_2 \\ v &= \sin\phi_1 \sin\theta_1 - \sin\phi_2 \sin\theta_2 \\ w &= -\cos\theta_1 - \cos\theta_2 \end{aligned}$$

Computing Shaders

Shader = computing Fourier transforms

I have done this for:

- Gaussian random surfaces
- Fractal random surfaces
- Periodic surfaces

Details in http://reality.sgi.com/jstam_sea/Research/ps/diff.ps.gz

Implementation

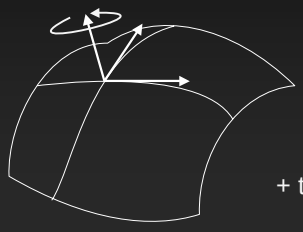
Implemented as MAYA plugin

$$I = |P(ku, kv)|^2$$

Straightforward

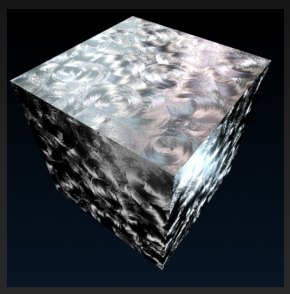
Implementation

Assign frame to the surface

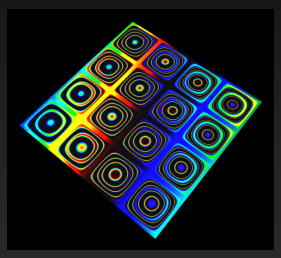


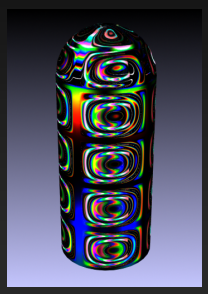
+ twist angle

Brushed Metal



Diffraction





Compact Disk

Used Physical Dimensions:

- bump height : 150 *nm*
- bump width : 500 *nm*
- separation between tracks : 2500 *nm*

Results

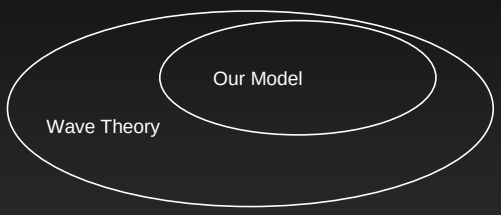
Animations Rendered in MAYA 2.0

Conclusion

Fourier transform very powerful tool
Most general Illumination model in CG
He-Torrance special case

Experimental validation (?)

Future Work



Multiple scattering, varying Fresnel coefficient, any distances, polarization, non-height field surfaces, subsurface scattering, etc.